

Anisotropic dynamical scaling in a spin model with competing interactions

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Results are presented for the kinetics of domain growth of a two-dimensional Ising spin model with competing interactions quenched from a disordered to a striped phase. The domain growth exponents are $\beta=1/2$ and $\beta=1/3$ for single-spin-flip and spin-exchange dynamics, as found in previous simulations. However, the correlation functions measured in the direction parallel and transverse to the stripes are different as suggested by the existence of different interface energies between the ground states of the model. In the case of single-spin-flip dynamics an anisotropic version of the Ohta-Jasnow-Kawasaki theory for the pair scaling function can be used to fit our data. [S1063-651X(97)13210-X]

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The kinetics of phase separation as systems with competing interactions quenched below their ordering temperature is the subject of much current research in a variety of fields in physics and other sciences [1]. Sometimes the equilibrium configuration of these systems is a lamellar structure with the ordered phases arranged in stripes. Our aim here is to study the growth of striped phases in a generalization of the two-dimensional (2D) Ising model. In particular, we will focus our attention on dynamical properties of the correlation functions in the directions parallel and transverse to the stripes.

In the simplest cases the late time ordering process is characterized by a single time dependent length $R(t) \sim t^\beta$ representing the average domain size [2]. This implies a particular behavior for the correlation function: if $\varphi(\vec{x}, t)$ is the ordering field the equal time correlation $C(\vec{r}, t) \equiv \langle \varphi(\vec{x}, t) \varphi(\vec{x} + \vec{r}, t) \rangle$ has the scaling form $C(\vec{r}, t) = f(r/R(t))$ where $f(z)$ is a scaling function. Our results for the striped phase show the same growth exponent in any direction relative to the stripes and an explicit dependence of the *different* longitudinal and transversal scaling functions on the microscopic details of the system. We will try to understand this difference and, in the case of single-spin-flip dynamics, we interpret our data by a generalization to anisotropic cases of the Ohta-Jasnow-Kawasaki (OJK) theory [3] for the scaling function. The anisotropic OJK theory is then tested on the Ising model with different nearest-neighbor couplings for the two square lattice directions.

Lamellar phases appear in many physical systems. Examples are diblock copolymer melts [4], surfactant-oil-water mixtures [5], and dipolar fluids with long-range Coulombic interactions [6]. Different from these systems, the striped phase of this paper is not characterized by any mesoscopic

length—the width of the lamellae is always one lattice spacing in the model considered here. Nevertheless our results may be relevant for systems with more realistic lamellar phases. Moreover, from a more theoretical point of view, it is interesting to have an explicit example of how the scaled correlation functions can depend on the details of the system, also in relation with recent discussions on this theme [7].

The model we consider is the well-known Ising version of the bidimensional isotropic eight vertex model [8], with Hamiltonian H given by

$$-\beta H = J_1 \sum_{\langle ij \rangle} s_i s_j + J_2 \sum_{\langle\langle ij \rangle\rangle} s_i s_j + J_3 \sum_{[i,j,k,l]} s_i s_j s_k s_l, \quad (1)$$

where s_i are Ising spins on a bidimensional square lattice and the sums are respectively on nearest neighbor pairs of spins, next to the nearest neighbor pairs and plaquettes. Periodic boundary conditions are always assumed. At $J_2 < 0, |J_1| < 2|J_2|$ and J_3 small a critical line separates the paramagnetic phase from a region where four phases corresponding to ground states with alternate plus and minus columns or rows coexist [8]. Sudden quenches of the system from a completely disordered initial configuration to the stripe-ordered phases will be studied by numerical simulations at both zero and finite temperature, for different values of the parameters, with heat-bath single-spin-flip [9] and spin-exchange dynamics [10].

Before presenting our results it is useful to summarize the known behavior of system (1) during the phase separation process. First consider quenching in the ferromagnetic phase in both $d=2$ and $d=3$. When all the couplings are ferromagnetic the growth exponent is $\beta=1/2$ or $\beta=1/3$ [2,11,12] corresponding respectively to single-spin-flip and spin-exchange dynamics. The situation becomes different for a weak antiferromagnetic coupling J_2 when, still in the ferromagnetic phase, energy barriers are opposite the coarsening of the domains and the system does not relax to equilibrium if the temperature is zero [13]. In $d=3$ the energy barriers are proportional to the linear dimension of domains and a logarithmic growth is expected [13]. Quenching in the striped phase in $d=2$ has been already studied in [14]. The

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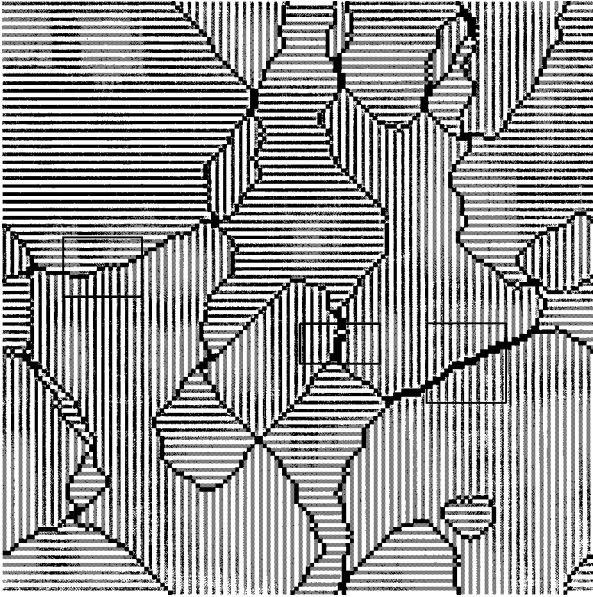


FIG. 1. Typical configuration of model (1) in the scaling regime. Gray and white squares represent, respectively, plus and minus spins; black squares represent the interface sites. The picture refers to a 100×100 square lattice, at zero temperature, with parameters $J_1=0.1$, $J_2=-1.0$, and $J_3=0$, after 150 MCS (Monte Carlo steps per site). The three boxes evidence different kinds of interfaces.

simulations of [14] show that also in the striped phase the average size of the domains grows as $R(t) \sim t^\beta$ with $\beta=1/2$ or $\beta=1/3$ for single-spin-flip and spin-exchange dynamics. Motivated by the idea of studying asymptotic correlations in lamellar phases we have analyzed again the growth and the dynamical scaling in the striped phase of model (1).

A typical configuration of the system during the evolution after a quench at $T=0$ is given in Fig. 1. The configuration is a patchwork of vertical and horizontal striped domains. Simulations show that in the case of single-spin-flip dynamics there are no energy barriers and the system separates also at zero temperature. To monitor the domain growth we evaluated the number of interfaces present in the system. To decide if a given site belongs to an interface or not, we compare the configuration of the system in a neighborhood of the site with four given patterns corresponding to the four ground states of the model. Then it is possible to define a distance between each pattern and the local configuration of the system. If this distance is greater than some threshold we say the site belongs to an interface, otherwise it is a part of some domain [15]. Our results have been shown not to depend on the values of the threshold and of the size of the patterns to be compared. Interfaces identified in this way are shown in Fig. 1. In $d=2$ the total length of interfaces per unit volume L scales as the inverse of the average size of domains. The results of [14] are confirmed by our simulations: we find $L \sim t^{-0.5}$ in the case of single-spin-flip dynamics and $L \sim t^{-1/3}$ in the spin-exchange dynamics. The same conclusions have been obtained also when monitoring the shrinking of a $N \times N$ square domain of one phase immersed in a sea of the three other phases. In this case the dependence on N of the time of shrinking can be used to evaluate the growth exponent.

We now turn to consider the correlation properties of

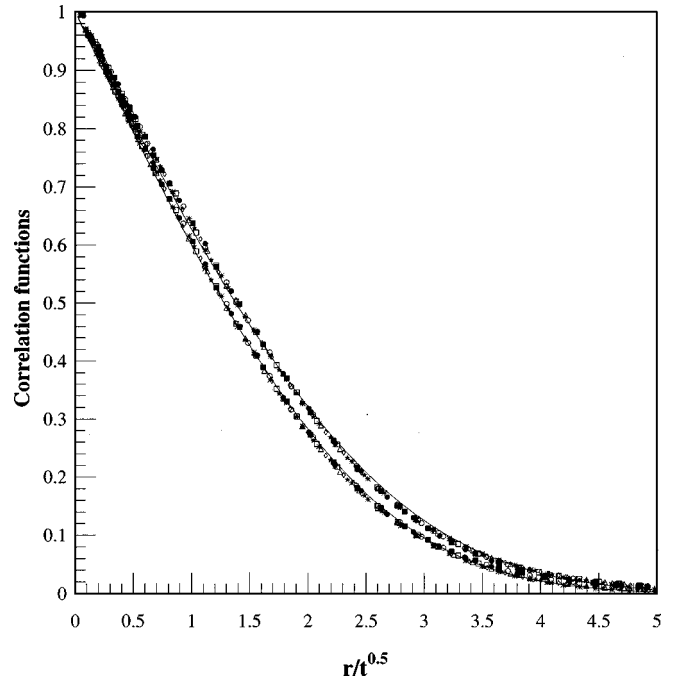


FIG. 2. Scaling functions $f_\ell(z)$ and $f_t(z)$, $z=r/t^{0.5}$ in the case of single-spin-flip dynamics with $T=0$, $J_1=0.1$, $J_2=-1$, and $J_3=0$. We averaged over 50 different histories on a 400×400 lattice. Longitudinal (above) and transversal (below) correlations are shown at times 180(●), 220(■), 260(○), 300(□), 340(△), 380(◇), 420(★), and 460(*). The solid lines are the best OJK fits.

growing domains in the scaling regime. We introduce the equal time *longitudinal* and *transversal* correlation functions respectively defined as

$$C_\ell(r,t) = \langle s(i,j)s(i+\epsilon(i,j)r, j+[1-\epsilon(i,j)]r) \rangle,$$

$$C_t(r,t) = \langle (-1)^r s(i,j)s(i+[1-\epsilon(i,j)]r, j+\epsilon(i,j)r) \rangle, \quad (2)$$

where $\epsilon(i,j)$ is 1 (zero) if site (i,j) belongs to a horizontal (vertical) domain and the average is performed over all the sites not belonging to interfaces at time t and over different stories of the system. These correlations have the property that for a fixed (i,j) they become zero outside a given domain. The scaling behavior of C_ℓ and C_t for a particular case with single-spin-flip dynamics is shown in Fig. 2 where data taken at different times have been plotted in terms of the scaling variable $z=r/\sqrt{t}$. The parameters of the simulations of Fig. 2 were $J_1=0.1$, $J_2=-1$, $J_3=0$. We see that the longitudinal and the transversal scaling functions f_ℓ and f_t are different, which is a general feature resulting from our simulations [16]. Actually the difference between f_ℓ and f_t is very tiny in the case of spin-exchange dynamics, but always understandable on the basis of the arguments given below. In the following we will show results only for simulations with single-spin-flip dynamics.

To explain the observed difference between f_ℓ and f_t a simple argument can help: a ferromagnetic coupling J_1 would favor longitudinal over transversal correlations while the reverse is true when J_1 is negative. This is indeed what happens. The results of simulations at $J_1=-0.1$, $J_2=-1$, $J_3=0$ are the same as those of Fig. 2 but with the role

of C_{\parallel} and C_t reversed. With fixed J_2 and J_3 , not only is C_{\parallel} always greater than C_t for $J_1 > 0$ while $C_{\parallel} < C_t$ for $J_1 < 0$, but the behavior of C_{\parallel} and C_t is symmetric with respect to the change of the sign of J_1 . A heuristic argument, based on the existence of interfaces with different surface tensions between the four different domains, can explain this symmetry. For simplicity, consider interfaces parallel to the lattice directions and $J_3 = 0$. There are three types of these interfaces all depicted in the boxes in Fig. 1. On the left of the picture a boundary between a vertical and a horizontal domain is marked; this interface reduces both longitudinal and transversal correlations and it does not matter for our argument. The situation is different for the two other kinds of interfaces marked in Fig. 1: the interface in the middle reduces only transversal correlations while the one on the right reduces only longitudinal correlations not influencing the transversal ones. The $T=0$ excess energy of the interface in the middle is $2J_2 - J_1$ while the excess energy for the interface on the right is $2J_2 + J_1$. Since the excess energy is the driving force for the phase separation process, if we assume that the role of these two interfaces is the same during this process, we can expect a symmetric behavior of the longitudinal and transversal correlation functions with respect to the change of sign of J_1 .

A theoretical prediction for the behavior of the pair correlation scaling function in models with nonconserved scalar field is given by the OJK theory [3]. Monte Carlo data have been shown to be reproduced by the OJK theory better than by other theories after an appropriate rescaling of the temporal coordinate [17]. The scaling function of the OJK theory is given by

$$f(z) = \frac{2}{\pi} \sin^{-1}[\exp(-z^2/D)], \quad (3)$$

where $z = r/t^{1/2}$ and $D = 8(d-1)/d$. This gives the Porod linear behavior [18] at small z of the correlation function $f(z) \sim 1 - \alpha z$ with $\alpha = 2\sqrt{2}/(\pi\sqrt{D})$. Practically, Monte Carlo and theoretical predictions can be compared imposing the same Porod behavior. This procedure in our simulations gives two different α for the longitudinal and transversal correlations, $\alpha_{\parallel} = 0.383$ and $\alpha_t = 0.414$ for $J_1 = 0.1$. The above discussed symmetry corresponds to the fact that when $J_1 = -0.1$ the best fit with the OJK function is given by $\alpha_{\parallel} = 0.406$ and $\alpha_t = 0.376$.

The above results show that the OJK theory well describes the pair scaling function in anisotropic cases if a free parameter is used to fit the data in the different directions. Since the surface tension is the origin of the anisotropic behavior, it is reasonable to check the validity of the OJK theory for anisotropic surface tension models in the simplest case corresponding to a field model with anisotropic kinetic terms. We consider the time dependent Ginzburg-Landau equation

$$\frac{\partial \varphi}{\partial t} = B_x \frac{\partial^2 \varphi}{\partial x^2} + B_y \frac{\partial^2 \varphi}{\partial y^2} - V'(\varphi), \quad (4)$$

where $V(\varphi)$ is the usual double-well potential. The rescaling $x \rightarrow x' = \sqrt{B_x}x$ and $y \rightarrow y' = \sqrt{B_y}y$ would formally eliminate

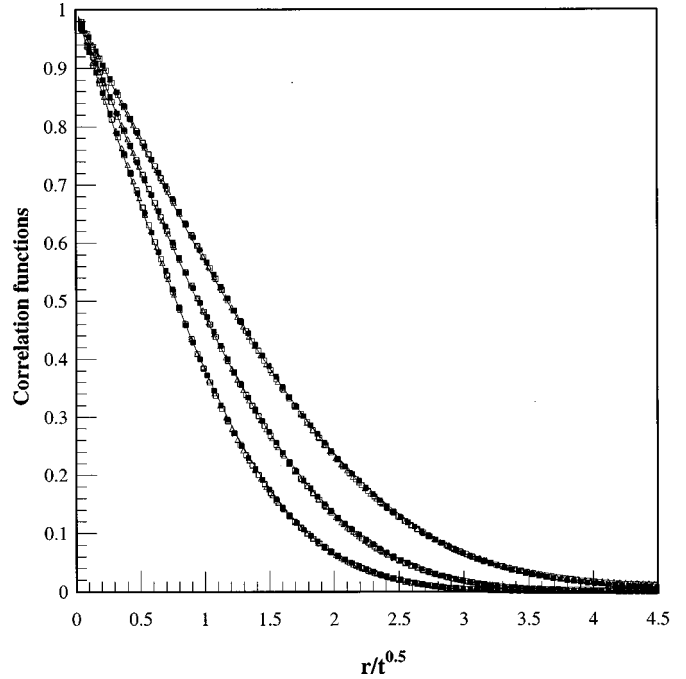


FIG. 3. Scaling collapse of the correlation functions of the Ising model at finite temperature. Data were obtained averaging over 250 stories in the anisotropic case, $J_x = 2$ and $J_y = 1$, and over 447 stories in the isotropic case, $J_x = J_y = 1$, on a 400×400 square system. From above to below, correlations along the x direction in the anisotropic case, correlations in the isotropic case, and correlation along the y direction in the anisotropic case are shown at times 350(■), 450(□), and 500(△). The solid lines are the best OJK fits.

the anisotropy and would give the usual spherically symmetric OJK theory. However, the correlations in terms of the original space coordinates would have different Porod laws with $\alpha_x/\alpha_y = \sqrt{B_y/B_x}$, showing an anisotropic behavior. Of course, it is not possible to simultaneously eliminate by a rescaling both B_x and B_y when higher order derivative terms are present in the dynamical equation, so that the scaling function in a general anisotropic model is expected to depend in some irreducible way on microscopic parameters.

The anisotropic OJK behavior for the model (4) can be tested by studying the dynamical scaling of the Ising model with different coupling J_x and J_y in the two lattice directions. In Fig. 3 the scaled correlation data in the x and the y directions are compared with the OJK scaling function. The simulations are at finite temperature with $J_x = 2$ and $J_y = 1$. Data for the usual Ising model with $J_x = J_y = 1$ are also plotted. The above rescaling argument would suggest $\alpha_x/\alpha_y = \sqrt{J_y/J_x}$. We measure a ratio $\alpha_y/\alpha_x = 1.548$, which is not far from the expected ratio 1.414 and confirms the fact that the OJK theory well reproduces the simulation data in anisotropic cases. The agreement with the theoretical expectation becomes stronger by decreasing the value of the temperature. In the case $J_x/J_y = 2$ and zero temperature we have measured $\alpha_y/\alpha_x = 1.4304$.

To conclude, we have studied domain growth in a spin model with four equivalent striped ground states [19]. The pair correlation functions are different when measured in the

direction parallel and transverse to the stripes. An explanation of this difference can be given on the basis of the different interface energies between the four ground states. Our results with single-spin-flip dynamics can be analyzed in the context of the Ohta-Jasnow-Kawasaki theory, which is expected to well reproduce scaling data in anisotropic surface

tension models. Whether other quantities such as the auto-correlation exponent or other correlation functions not considered here depend on the anisotropy of the system is a matter for a future study.

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